

$$V = \frac{1}{3} \pi \left(\frac{25h^2}{196} \right) h = \frac{25\pi}{588} \cdot h^3$$

$$\frac{dV}{dt} = \frac{25\pi}{588} \cdot 3h^2 \cdot \frac{dh}{dt} = \frac{25\pi}{196} h^2 \left[\frac{dh}{dt} \right]$$

$$\text{At } h=6; \text{ set } -2 = \frac{25\pi}{196} (36) \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{-2 \times 196}{25\pi \times 36} = \frac{-98}{225\pi} \approx -0.1356.$$

(b) Asking for $\frac{dr}{dt}$

$$h = \frac{14}{5} r \Leftrightarrow r = \frac{5h}{14}$$

$$\frac{dr}{dt} = \frac{5}{14} \cdot \frac{dh}{dt} = \frac{5}{14} \left(\frac{-98}{225\pi} \right) = \frac{-7}{45\pi} //$$

Optimization: (A) Typically, goal is to "optimize" $\begin{cases} \rightarrow \text{max something good} \\ \rightarrow \text{min} - \text{bad} \end{cases}$

(B) want to achieve this goal within a special set of constants

- Identify goal
- Write down the set of constraints clearly
- Sketch a model
- Formulate a math model, solve using calculus

Ex) Suppose that a ball is thrown up in the air and its height after t seconds is $4 + 48t - 16t^2$ ft. Determine how long it will take for the ball to reach its maximum height and determine this max height.

